# Listing the solutions in the interval

A trigonometric identity is an equation that's true of all angles at which the expressions on both sides (of the equation) are defined. In many situations, however, you'll be dealing with an equation that's true of only certain angles, and you'll need to solve such an equation to determine the set of angles that satisfy that equation. Trigonometric identities will often come in handy in solving such equations.

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**Example**

Find all the values of $\theta$ that make $\sin(2\theta) = \sin\theta$ a true equation.

Change the left side of the equation using the double-angle identity $\sin(2\theta) = 2\sin\theta\cos\theta$.

$$\sin(2\theta) = \sin\theta$$

$$2\sin\theta\cos\theta = \sin\theta$$

Subtract $\sin\theta$ from both sides, then factor and solve.

$$2\sin\theta\cos\theta - \sin\theta = 0$$

$$\sin\theta(2\cos\theta - 1) = 0$$

$\sin\theta = 0$ and $2\cos\theta - 1 = 0$

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The equation $\sin\theta = 0$ is true when $\theta = 0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ \ldots$. We can rewrite this as

$$\theta = 0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ \ldots$$

$$\theta = \pi\,(0,\ 1,\ 2,\ 3,\ 4,\ \ldots)$$

$$\theta = n\pi \quad \text{where} \quad n = 0,\ 1,\ 2,\ 3,\ 4,\ \ldots$$

The equation $2\cos\theta - 1 = 0$ is true when

$$2\cos\theta - 1 = 0$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

So $\theta$ is a solution of $2\cos\theta - 1 = 0$ when

$$\theta = \frac{\pi}{3} + 2n\pi$$

$$\theta = \frac{5\pi}{3} + 2n\pi$$

Therefore, $\sin(2\theta) = \sin\theta$ is true when

$$\theta = n\pi$$

$$\theta = \frac{\pi}{3} + 2n\pi$$

$$\theta = \frac{5\pi}{3} + 2n\pi$$

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Sometimes you'll need to list only the solutions that lie inside the interval $[0,2\pi]$. In the equation $\sin(2\theta) = \sin\theta$, the angles $\theta$ that are in the solution set and lie in the interval $[0,2\pi]$ are $0$, $\pi$, $\pi/3$, and $5\pi/3$.

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In some cases, you may have to find the roots of a polynomial to get the solutions of a trigonometric equation.

**Example**

Find all the solutions of the equation $\tan^2\theta + 2\tan\theta + 1 = 0$, and list the ones that lie in the interval $[0,2\pi]$.

Let's replace $\tan\theta$ with $u$. Then we have the equation

$$u^2 + 2u + 1 = 0$$

We can factor $u^2 + 2u + 1$ as $(u+1)^2$. Therefore, we need to solve the equation

$$(\tan\theta + 1)^2 = 0$$

$$\tan\theta + 1 = 0$$

$$\tan\theta = -1$$

We know that $\tan(3\pi/4) = -1$ and $\tan(7\pi/4) = -1$. Also, every angle that's coterminal with an angle of either $3\pi/4$ or $-\pi/4$ has a tangent of $-1$, and those are all of the solutions. So $\theta$ is a solution of $\tan^2\theta + 2\tan\theta + 1 = 0$ when

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$$\theta = \frac{3\pi}{4} + 2n\pi$$

or

$$\theta = \frac{7\pi}{4} + 2n\pi$$

The only angles $\theta$ that are solutions of the equation $\tan^2\theta + 2\tan\theta + 1 = 0$ and lie in the interval $[0,2\pi]$ are $\theta = 3\pi/4$ and $\theta = 7\pi/4$.

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In the next example, we'll use some trig identities.

**Example**

Find all the solutions of the equation and list the solutions that lie in the interval $[0,2\pi)$.

$$\sin^2\theta + \cos(2\theta) = 1$$

By the half-angle formula for sine applied to the angle $\theta$, we get

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

Therefore,

$$\sin^2\theta + \cos(2\theta) = \frac{1 - \cos(2\theta)}{2} + \cos(2\theta)$$

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Using a common denominator and then simplifying, we get

$$\sin^2\theta + \cos(2\theta) = \frac{1 - \cos(2\theta) + 2[\cos(2\theta)]}{2}$$

$$\sin^2\theta + \cos(2\theta) = \frac{1 + \cos(2\theta)}{2}$$

By the half-angle formula for cosine applied to the angle $\theta$,

$$\frac{1 + \cos(2\theta)}{2} = \cos^2\theta$$

Thus

$$\sin^2\theta + \cos(2\theta) = \cos^2\theta$$

What this means is that we're setting out to find the solutions of the equation

$$\cos^2\theta = 1$$

$$\cos\theta = \pm 1$$

As we know, $\cos\theta = 1$ at $\theta = 0,\ 2\pi,\ 4\pi,\ 6\pi,\ \ldots$. Similarly, $\cos\theta = -1$ at $\theta = \pi,\ 3\pi,\ 5\pi,\ 7\pi,\ \ldots$. Combining these results, $\cos^2\theta = 1$ when $\theta$ is an integer multiple of $\pi$.

The only integer multiples of $\pi$ in the interval $[0,2\pi)$ are $0$ and $\pi$. So the only solutions of $\sin^2\theta + \cos(2\theta) = 1$ in the interval are $\theta = 0$ and $\theta = \pi$.

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